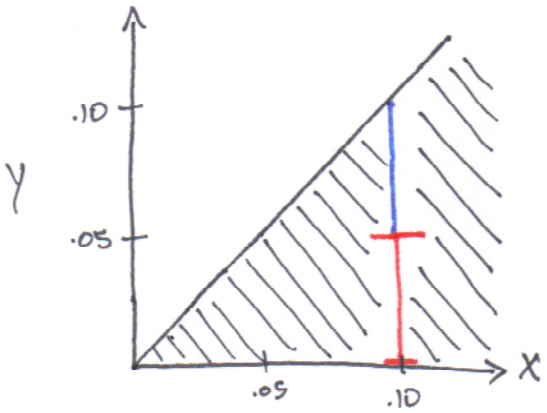
Problem 112

X: proportion who buy basic policy

Y: proportion who buy supplemental policy

$f(x,y) = 2(x+y) \qquad 0 \le y \le x \le 1$

- Proportions must be between 0 and 1
- Must buy basic before supplemental



$P(Y<.05 \mid X=.10)$

$f_{X,Y}(.10, y) = 2(.10+y) \qquad 0 \le y \le .10$

$f_X(.10) = \int_0^{.10} f_{X,Y}(.10, y) \cdot dy$

$= \int_0^{.10} 2(.10+y) \cdot dy = .20y + y^2 \Big|_0^{.10} = .20(.10) + (.10)^2 = .03$

$f(y \mid X=.10) = \frac{2(.10+y)}{.03} = \frac{200}{3}(.10+y) \qquad 0 \le y \le .10$

$P(Y<.05 \mid X=.10) = \int_0^{.05} f(y \mid X=.10) \cdot dy$

$= \int_0^{.05} \frac{200}{3}(.10+y) \cdot dy$

$= \frac{200}{3}(.10)y + \frac{100}{3}y^2 \Big|_0^{.05}$

$= \frac{200}{3}(.10)(.05) + \frac{100}{3}(.05)^2 = .41667$

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