

Problem 125

$$Y|X \sim \text{Uniform}[0, x] \Rightarrow f(y|x) = \frac{1}{x} \quad 0 < y < x$$
$$f(x) = 2x \quad 0 < x < 1$$

$$f(x|y) = \frac{f(x, y)}{f(y)}$$

$$f(x, y) = f(y|x) \cdot f(x) = 2 \quad 0 < y < x < 1$$
$$f(y) = \int_y^1 f(x, y) \cdot dx$$
$$= \int_y^1 2 \cdot dx = 2 - 2y$$

$$f(x|y) = \frac{2}{2-2y} = \frac{1}{1-y} \quad \boxed{E}$$