

Problem 136

X : number of rolls needed to obtain a 5

Y : number of rolls needed to obtain a 6

$$E(X|Y=2)$$

Strategy:

Adjust distribution of X due to condition $Y=2$

$Y=2 \Rightarrow$ First roll NOT a 6 $\{1,2,3,4,5\}$
Second roll IS a 6 $\{6\}$

$$p(X=1|Y=2) = \frac{1}{5}$$
$$p(X=2|Y=2) = 0$$

$$p(X>2|Y=2) = 1 - p(X=1|Y=2)$$
$$= 1 - \frac{1}{5} = \frac{4}{5}$$

# rolls	$p(X Y=2)$
1	$\frac{1}{5}$
2	0
3+	$\frac{4}{5}$

$$E(X|X>2) \longleftrightarrow 3+$$

$X \sim \text{Geometric}(p = \frac{1}{6})$

Z = number of trials before first success
 \Rightarrow success on Z^{th} trial

$$E(Z) = \frac{1}{p}$$

$$E(X) = 6$$

$$E(X|X>2) = E(X) + 2$$
$$= 6 + 2 = 8$$

$$E(X|Y=2) = \sum_{\# = 1}^{\infty} (\# \text{ rolls}) \cdot p(X|Y=2)$$

$$= 1\left(\frac{1}{5}\right) + 2(0) + 8\left(\frac{4}{5}\right)$$

$$= 6.6$$

