

Problem 145

$$f(y|x=.75) = \frac{f(.75, y)}{f_x(.75)}$$

$$f(.75, y) = \begin{cases} 1.5 & 0 < y < .5 \\ .75 & .5 < y < 1 \end{cases}$$

$$f_x(.75) = \int_0^{.5} f(.75, y) \cdot dy + \int_{.5}^1 f(.75, y) \cdot dy \\ = .5(1.5) + .5(.75) = 1.125$$

$$f(y|x=.75) = \begin{cases} \frac{4}{3} & 0 < y < .5 \\ \frac{2}{3} & .5 < y < 1 \end{cases}$$

$$E(y^2|x=.75) = \int_0^{.5} y^2 \cdot f(y|x=.75) \cdot dy + \int_{.5}^1 y^2 \cdot f(y|x=.75) \cdot dy \\ = \frac{4}{3} \int_0^{.5} y^2 \cdot dy + \frac{2}{3} \int_{.5}^1 y^2 \cdot dy \\ = \frac{4}{9} \cdot y^3 \Big|_0^{.5} + \frac{2}{9} \cdot y^3 \Big|_{.5}^1 = \frac{1}{4}$$

$$E(y|x=.75) = \frac{4}{3} \int_0^{.5} y \cdot dy + \frac{2}{3} \int_{.5}^1 y \cdot dy \\ = \frac{2}{3} \cdot y^2 \Big|_0^{.5} + \frac{1}{3} \cdot y^2 \Big|_{.5}^1 = \frac{5}{12}$$

$$V(y|x=.75) = \frac{1}{4} - \left(\frac{5}{12}\right)^2 \\ = \underline{\underline{.076}} \quad \boxed{C}$$