

Exam P 147

Solution:

Denote X - amount of a claim before deductible. Y - amount of a claim after deductible.

$$X \sim \text{exponential}(\lambda) \quad E(X) = \frac{1}{\lambda} \quad \text{Var}(X) = \frac{1}{\lambda^2}$$

$$E(X^2) = (E(X))^2 + \text{Var}(X) = \frac{2}{\lambda^2}$$

$$Y = \begin{cases} \text{not observed} & X \leq d \\ X - d & X > d \end{cases}$$

$$E(Y) = 0.9 E(X)$$

$$= 0.9 \frac{1}{\lambda}$$

By memoryless property of exponential dist'n

$$E(Y) = E(X - d \mid X > d)$$

$$= E(X \mid X > d) - E(d \mid X > d)$$

=

$$E(Y) = E(X - d \mid X > d)$$

$$= E(X) \cdot \Pr(X > d)$$

$$= \frac{1}{\lambda} \cdot \Pr(X > d)$$

$$= 0.9 \frac{1}{\lambda}$$

$$\Rightarrow \Pr(X > d) = 0.9$$

$$\begin{aligned} E(Y^2) &= E(X^2) \cdot \Pr(X > d) \\ &= 2\lambda^2 \cdot 0.9 \\ &= 1.8\lambda^2 \end{aligned}$$

$$\text{Var}(Y) = E(Y^2) - (E(Y))^2 = 1.8\lambda^2 - 0.81\lambda^2 = 0.99\lambda^2$$

$$\text{Var}(X) = \lambda^2$$

$$\frac{\lambda^2 - 0.99\lambda^2}{\lambda^2} = 0.01 = 1\%$$

choose A