

Exam P

Problem 151

Given 27 luggage in total, choose 4 of them.

The probability that 1 is damaged
 $= 2 \times$ none of the 4 is damaged.

Prob(2 of 4 is damaged) = ?

Key = Find something which is constant all the time.

↓
 The # of damaged pieces stays constant, denoted as r

$$\text{Prob}(1 \text{ out of } 4 \text{ is damaged}) \\ = \frac{\binom{r}{1} \binom{27-r}{3}}{\binom{27}{4}}$$

$$\text{Prob}(\text{None out of } 4 \text{ is damaged}) \\ = \frac{\binom{r}{0} \binom{27-r}{4}}{\binom{27}{4}}$$

$$\frac{\binom{r}{1} \binom{27-r}{3}}{\binom{27}{4}} = \frac{2 \cdot \binom{r}{0} \binom{27-r}{4}}{\binom{27}{4}}$$

$$r \cdot \frac{(27-r)!}{(24-r)! \cdot 3!} = 2 \cdot \frac{(27-r)!}{(23-r)! \cdot 4!}$$

$$(24-r)! = (24-r) \cdot (23-r)!$$

$$4r = 48 - 2r$$

$$r = 8$$

Prob (Exactly 2 out of 4 are damaged)

$$= \frac{\binom{r}{2} \binom{27-r}{2}}{\binom{27}{4}}$$

$$= \frac{\binom{8}{2} \binom{19}{2}}{\binom{27}{4}}$$

$$= \frac{266}{975} = 0.27$$

choose C