Problem 151

Given 27 luggage in total, choose 4 of them.

The probability that 1 is damaged

\[ = 2 \times \text{none of the 4 is damaged}. \]

\[ \text{Prob}(2 \text{ of } 4 \text{ is damaged}) = ? \]

key: Find something which is constant all the time.

The # of damaged pieces stays constant, denoted as \( r \).

\[ \text{Prob}(1 \text{ out of } 4 \text{ is damaged}) = \frac{\binom{r}{1} \binom{27-r}{3}}{\binom{27}{4}} \]

\[ \text{Prob}(\text{None out of } 4 \text{ is damaged}) = \frac{\binom{r}{0} \binom{27-r}{4}}{\binom{27}{4}} \]

\[ \frac{\binom{r}{1} \binom{27-r}{3}}{\binom{27}{4}} = 2 \cdot \frac{\binom{r}{0} \binom{27-r}{4}}{\binom{27}{4}} \]

\[ r \cdot \frac{(27-r)!}{(24-r)! \cdot 3!} = 2 \cdot \frac{(27-r)!}{(23-r)! \cdot 4!} \]

\[ (24-r)! = (24 \cdot r) \cdot (23-r)! \]

\[ 4r = 48 - 2r \cdot \]

\[ r = 8 \]
\[ \text{Prob( exactly 2 out of 4 are damaged)} \]
\[ = \frac{\binom{r}{2} \binom{27-r}{2}}{\binom{27}{4}} \]
\[ = \frac{\binom{8}{2} \binom{19}{2}}{\binom{27}{4}} \]
\[ = \frac{266}{975} \approx 0.27 \]

choose C