

42. Solution: D

Let

$I_A$  = Event that Company A makes a claim

$I_B$  = Event that Company B makes a claim

$X_A$  = Expense paid to Company A if claims are made

$X_B$  = Expense paid to Company B if claims are made

Then we want to find

$$\begin{aligned}\Pr\left\{\left[I_A^C \cap I_B\right] \cup \left[\left(I_A \cap I_B\right) \cap \left(X_A < X_B\right)\right]\right\} \\ &= \Pr\left[I_A^C \cap I_B\right] + \Pr\left[\left(I_A \cap I_B\right) \cap \left(X_A < X_B\right)\right] \\ &= \Pr\left[I_A^C\right] \Pr\left[I_B\right] + \Pr\left[I_A\right] \Pr\left[I_B\right] \Pr\left[X_A < X_B\right] \quad (\text{independence}) \\ &= (0.60)(0.30) + (0.40)(0.30) \Pr\left[X_B - X_A \geq 0\right] \\ &= 0.18 + 0.12 \Pr\left[X_B - X_A \geq 0\right]\end{aligned}$$

Now  $X_B - X_A$  is a linear combination of independent normal random variables.

Therefore,  $X_B - X_A$  is also a normal random variable with mean

$$M = E\left[X_B - X_A\right] = E\left[X_B\right] - E\left[X_A\right] = 9,000 - 10,000 = -1,000$$

and standard deviation  $\sigma = \sqrt{\text{Var}\left(X_B\right) + \text{Var}\left(X_A\right)} = \sqrt{(2000)^2 + (2000)^2} = 2000\sqrt{2}$

It follows that

$$\begin{aligned}\Pr\left[X_B - X_A \geq 0\right] &= \Pr\left[Z \geq \frac{1000}{2000\sqrt{2}}\right] \quad (Z \text{ is standard normal}) \\ &= \Pr\left[Z \geq \frac{1}{2\sqrt{2}}\right] \\ &= 1 - \Pr\left[Z < \frac{1}{2\sqrt{2}}\right] \\ &= 1 - \Pr\left[Z < 0.354\right] \\ &= 1 - 0.638 = 0.362\end{aligned}$$

Finally,

$$\begin{aligned}\Pr\left\{\left[I_A^C \cap I_B\right] \cup \left[\left(I_A \cap I_B\right) \cap \left(X_A < X_B\right)\right]\right\} &= 0.18 + (0.12)(0.362) \\ &= 0.223\end{aligned}$$