47. Solution: D

Let T be the time from purchase until failure of the equipment. We are given that T is exponentially distributed with parameter $\lambda = 10$ since $10 = E[T] = \lambda$. Next define the payment

P under the insurance contract by $P = \begin{cases} x & \text{for } 0 \le T \le 1 \\ \frac{x}{2} & \text{for } 1 < T \le 3 \\ 0 & \text{for } T > 3 \end{cases}$

We want to find x such that

$$1000 = E[P] = \int_0^1 \frac{x}{10} e^{-t/10}\,dt + \int_1^3 \frac{x}{2}\frac{1}{10} e^{-t/10}\,dt = -xe^{-t/10}\Big|_0^1 - \frac{x}{2}e^{-t/10}\Big|_1^3$$

$$= -x\,e^{-1/10} + x - (x/2)\,e^{-3/10} + (x/2)\,e^{-1/10} = x(1 - \tfrac{1}{2}\,e^{-1/10} - \tfrac{1}{2}\,e^{-3/10}) = 0.1772x\,.$$

We conclude that $x = 5644$.