47. Solution: D
Let $T$ be the time from purchase until failure of the equipment. We are given that $T$ is exponentially distributed with parameter $\lambda = 10$ since $10 = E[T] = \frac{1}{\lambda}$. Next define the payment

$$P = \begin{cases} x & \text{for } 0 \leq T \leq 1 \\ \frac{x}{2} & \text{for } 1 < T \leq 3 \\ 0 & \text{for } T > 3 \end{cases}$$

We want to find $x$ such that

$$1000 = E[P] = \frac{1}{10} \int_0^1 x e^{-t/10} dt + \frac{3}{10} \int_1^3 \frac{1}{2} x e^{-t/10} dt = -x e^{-t/10} \bigg|_0^1 + \frac{x}{2} e^{-t/10} \bigg|_1^3$$

$$= -x e^{-1/10} + x - \frac{x}{2} e^{-3/10} + \frac{x}{2} e^{-1/10} = x\left(1 - \frac{1}{2} e^{-1/10} - \frac{1}{2} e^{-3/10}\right) = 0.1772x$$

We conclude that $x = 5644$.

48. Solution: E
Let $X$ and $Y$ denote the year the device fails and the benefit amount, respectively. Then the density function of $X$ is given by

$$f_X(x) = \begin{cases} 0.6 & \text{if } x = 1, 2, 3, 4 \\ 0.4 & \text{if } x = 4, 5 \end{cases}$$

and

$$y = \begin{cases} 1000(5-x) & \text{if } x = 1, 2, 3, 4 \\ 0 & \text{if } x > 4 \end{cases}$$

It follows that

$$E[Y] = 4000(0.4) + 3000(0.6)(0.4) + 2000(0.6)^2(0.4) + 1000(0.6)^3(0.4)$$

$$= 2694$$

49. Solution: D
Define $f(X)$ to be hospitalization payments made by the insurance policy. Then

$$f(X) = \begin{cases} 100X & \text{if } X = 1, 2, 3 \\ 300 + 25(X - 3) & \text{if } X = 4, 5 \end{cases}$$

and