

55. Solution: C

The pdf of x is given by $f(x) = \frac{k}{(1+x)^4}$, $0 < x < \infty$. To find k , note

$$1 = \int_0^{\infty} \frac{k}{(1+x)^4} dx = -\frac{k}{3} \frac{1}{(1+x)^3} \Big|_0^{\infty} = \frac{k}{3}$$

$$k = 3$$

It then follows that $E[x] = \int_0^{\infty} \frac{3x}{(1+x)^4} dx$ and substituting $u = 1 + x$, $du = dx$, we see

$$E[x] = \int_1^{\infty} \frac{3(u-1)}{u^4} du = 3 \int_1^{\infty} (u^{-3} - u^{-4}) du = 3 \left[\frac{u^{-2}}{-2} - \frac{u^{-3}}{-3} \right]_1^{\infty} = 3 \left[\frac{1}{2} - \frac{1}{3} \right] = 3/2 - 1 = 1/2.$$