

62. Solution: C

First note that the density function of X is given by

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } x=1 \\ x-1 & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Then

$$\begin{aligned} E(X) &= \frac{1}{2} + \int_1^2 x(x-1) dx = \frac{1}{2} + \int_1^2 (x^2 - x) dx = \frac{1}{2} + \left(\frac{1}{3}x^3 - \frac{1}{2}x^2 \right) \Big|_1^2 \\ &= \frac{1}{2} + \frac{8}{3} - \frac{4}{2} - \frac{1}{3} + \frac{1}{2} = \frac{7}{3} - 1 = \frac{4}{3} \end{aligned}$$

$$\begin{aligned} E(X^2) &= \frac{1}{2} + \int_1^2 x^2(x-1) dx = \frac{1}{2} + \int_1^2 (x^3 - x^2) dx = \frac{1}{2} + \left(\frac{1}{4}x^4 - \frac{1}{3}x^3 \right) \Big|_1^2 \\ &= \frac{1}{2} + \frac{16}{4} - \frac{8}{3} - \frac{1}{4} + \frac{1}{3} = \frac{17}{4} - \frac{7}{3} = \frac{23}{12} \end{aligned}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{23}{12} - \left(\frac{4}{3} \right)^2 = \frac{23}{12} - \frac{16}{9} = \frac{5}{36}$$