66. Solution: E

Let X_1, X_2, X_3, and X_4 denote the four independent bids with common distribution function F. Then if we define $Y = \max(X_1, X_2, X_3, X_4)$, the distribution function G of Y is given by

$$\begin{aligned} G(y) &= \Pr[Y \le y] \\ &= \Pr[(X_1 \le y) \cap (X_2 \le y) \cap (X_3 \le y) \cap (X_4 \le y)] \\ &= \Pr[X_1 \le y]\Pr[X_2 \le y]\Pr[X_3 \le y]\Pr[X_4 \le y] \\ &= [F(y)]^4 \\ &= \frac{1}{16}(1+\sin\pi y)^4 \quad , \quad \frac{3}{2} \le y \le \frac{5}{2} \end{aligned}$$

It then follows that the density function g of Y is given by

$$\begin{aligned} g(y) &= G'(y) \\ &= \frac{1}{4}(1+\sin\pi y)^3(\pi\cos\pi y) \\ &= \frac{\pi}{4}\cos\pi y(1+\sin\pi y)^3 \quad , \quad \frac{3}{2} \le y \le \frac{5}{2} \end{aligned}$$

Finally,

$$\begin{aligned} E[Y] &= \int_{3/2}^{5/2} yg(y)\,dy \\ &= \int_{3/2}^{5/2} \frac{\pi}{4} y\cos\pi y(1+\sin\pi y)^3\,dy \end{aligned}$$