76. Solution: A

First, observe that the distribution function of X is given by

$$F(x) = \int_1^x \frac{3}{t^4}\,dt = -\frac{1}{t^3}\Big|_1^x = 1 - \frac{1}{x^3} \quad , \quad x > 1$$

Next, let X_1, X_2, and X_3 denote the three claims made that have this distribution. Then if Y denotes the largest of these three claims, it follows that the distribution function of Y is given by

$$G(y) = \Pr[X_1 \le y]\Pr[X_2 \le y]\Pr[X_3 \le y]$$

$$= \left(1 - \frac{1}{y^3}\right)^3 \quad , \quad y > 1$$

while the density function of Y is given by

$$g(y) = G'(y) = 3\left(1 - \frac{1}{y^3}\right)^2\left(\frac{3}{y^4}\right) = \left(\frac{9}{y^4}\right)\left(1 - \frac{1}{y^3}\right)^2 \quad , \quad y > 1$$

Therefore,

$$E[Y] = \int_1^\infty \frac{9}{y^3}\left(1 - \frac{1}{y^3}\right)^2 dy = \int_1^\infty \frac{9}{y^3}\left(1 - \frac{2}{y^3} + \frac{1}{y^6}\right) dy$$

$$= \int_1^\infty \left(\frac{9}{y^3} - \frac{18}{y^6} + \frac{9}{y^9}\right) dy = \left[-\frac{9}{2y^2} + \frac{18}{5y^5} - \frac{9}{8y^8}\right]_1^\infty$$

$$= 9\left[\frac{1}{2} - \frac{2}{5} + \frac{1}{8}\right] = 2.025 \text{ (in thousands)}$$