86. Solution: E

Let $X_1, \ldots, X_{100}$ denote the number of pensions that will be provided to each new recruit. Now under the assumptions given,

$$X_i = \begin{cases} 0 & \text{with probability} \quad 1-0.4=0.6 \\ 1 & \text{with probability} \quad (0.4)(0.25)=0.1 \\ 2 & \text{with probability} \quad (0.4)(0.75)=0.3 \end{cases}$$

for $i = 1, \ldots, 100$. Therefore,

$$E[X_i] = (0)(0.6) + (1)(0.1) + (2)(0.3) = 0.7 \,,$$

$$E\left[X_i^2\right] = (0)^2(0.6) + (1)^2(0.1) + (2)^2(0.3) = 1.3 \,, \quad \text{and}$$

$$\operatorname{Var}[X_i] = E\left[X_i^2\right] - \{E[X_i]\}^2 = 1.3 - (0.7)^2 = 0.81$$

Since $X_1, \ldots, X_{100}$ are assumed by the consulting actuary to be independent, the Central Limit Theorem then implies that $S = X_1 + \ldots + X_{100}$ is approximately normally distributed with mean

$$E[S] = E[X_1] + \ldots + E[X_{100}] = 100(0.7) = 70$$

and variance

$$\operatorname{Var}[S] = \operatorname{Var}[X_1] + \ldots + \operatorname{Var}[X_{100}] = 100(0.81) = 81$$

Consequently,

$$\begin{aligned} \Pr[S \le 90.5] &= \Pr\left[\frac{S-70}{9} \le \frac{90.5-70}{9}\right] \\ &= \Pr[Z \le 2.28] \\ &= 0.99 \end{aligned}$$