

104. Solution: B

Let us first determine k :

$$1 = \int_0^1 \int_0^1 kx dx dy = \int_0^1 \frac{1}{2} kx^2 \Big|_0^1 dy = \int_0^1 \frac{k}{2} dy = \frac{k}{2}$$

$$k = 2$$

Then

$$E[X] = \int_0^1 \int_0^1 2x^2 dy dx = \int_0^1 2x^2 dx = \frac{2}{3} x^3 \Big|_0^1 = \frac{2}{3}$$

$$E[Y] = \int_0^1 \int_0^1 y \cdot 2x dx dy = \int_0^1 y dy = \frac{1}{2} y^2 \Big|_0^1 = \frac{1}{2}$$

$$\begin{aligned} E[XY] &= \int_0^1 \int_0^1 2x^2 y dx dy = \int_0^1 \frac{2}{3} x^3 y \Big|_0^1 dy = \int_0^1 \frac{2}{3} y dy \\ &= \frac{2}{6} y^2 \Big|_0^1 = \frac{2}{6} = \frac{1}{3} \end{aligned}$$

$$\text{Cov}[X, Y] = E[XY] - E[X]E[Y] = \frac{1}{3} - \left(\frac{2}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{3} - \frac{1}{3} = 0$$

(Alternative Solution)

Define $g(x) = kx$ and $h(y) = 1$. Then

$$f(x, y) = g(x)h(y)$$

In other words, $f(x, y)$ can be written as the product of a function of x alone and a function of y alone. It follows that X and Y are independent. Therefore, $\text{Cov}[X, Y] = 0$.