

106.

Solution: C

The joint pdf of X and Y is $f(x,y) = f_2(y|x) f_1(x)$
 $= (1/x)(1/12)$, $0 < y < x$, $0 < x < 12$.

Therefore,

$$E[X] = \int_0^{12} \int_0^x x \cdot \frac{1}{12x} dy dx = \int_0^{12} \frac{y}{12} \Big|_0^x dx = \int_0^{12} \frac{x}{12} dx = \frac{x^2}{24} \Big|_0^{12} = 6$$

$$E[Y] = \int_0^{12} \int_0^x \frac{y}{12x} dy dx = \int_0^{12} \left[\frac{y^2}{24x} \right]_0^x dx = \int_0^{12} \frac{x}{24} dx = \frac{x^2}{48} \Big|_0^{12} = \frac{144}{48} = 3$$

$$E[XY] = \int_0^{12} \int_0^x \frac{y}{12} dy dx = \int_0^{12} \left[\frac{y^2}{24} \right]_0^x dx = \int_0^{12} \frac{x^2}{24} dx = \frac{x^3}{72} \Big|_0^{12} = \frac{(12)^3}{72} = 24$$

$$\text{Cov}(X,Y) = E[XY] - E[X]E[Y] = 24 - (3)(6) = 24 - 18 = 6.$$