107. Solution: A

$$\begin{aligned}\text{Cov}(C_1, C_2) &= \text{Cov}(X+Y,\ X+1.2Y)\\ &= \text{Cov}(X,X)+\text{Cov}(Y,X)+\text{Cov}(X,1.2Y)+\text{Cov}(\text{Y},1.2\text{Y})\\ &= \text{Var}\,X+\text{Cov}(X,Y)+1.2\text{Cov}(X,Y)+1.2\text{Var}Y\\ &= \text{Var}\,X+2.2\,\text{Cov}(X,Y)+1.2\text{Var}Y\end{aligned}$$

$$\text{Var}\,X = E(X^2)-(E(X))^2 = 27.4-5^2 = 2.4$$

$$\text{Var}\,Y = E(Y^2)-(E(Y))^2 = 51.4-7^2 = 2.4$$

$$\text{Var}(X+Y) = \text{Var}\,X+\text{Var}\,Y+2\,\text{Cov}(X,Y)$$

$$\text{Cov}(X,Y) = \frac{1}{2}(\text{Var}(X+Y)-\text{Var}\,X-\text{Var}\,Y) = \frac{1}{2}(8-2.4-2.4) = 1.6$$

$$\text{Cov}(C_1,C_2) = 2.4+2.2(1.6)+1.2(2.4) = 8.8$$

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107. Alternate solution:

We are given the following information:

$$C_1 = X+Y$$
$$C_2 = X+1.2Y$$
$$E[X]=5$$
$$E[X^2]=27.4$$
$$E[Y]=7$$
$$E[Y^2]=51.4$$
$$\text{Var}[X+Y]=8$$

Now we want to calculate

$$\begin{aligned}\text{Cov}(C_1,C_2) &= \text{Cov}(X+Y, X+1.2Y)\\ &= E[(X+Y)(X+1.2Y)]-E[X+Y]\bullet E[X+1.2Y]\\ &= E[X^2+2.2XY+1.2Y^2]-(E[X]+E[Y])(E[X]+1.2E[Y])\\ &= E[X^2]+2.2E[XY]+1.2E[Y^2]-(5+7)(5+(1.2)7)\\ &= 27.4+2.2E[XY]+1.2(51.4)-(12)(13.4)\\ &= 2.2E[XY]-71.72\end{aligned}$$

Therefore, we need to calculate $E[XY]$ first. To this end, observe

$$\begin{aligned}8 = \text{Var}[X+Y] &= E[(X+Y)^2]-(E[X+Y])^2\\ &= E[X^2+2XY+Y^2]-(E[X]+E[Y])^2\\ &= E[X^2]+2E[XY]+E[Y^2]-(5+7)^2\\ &= 27.4+2E[XY]+51.4-144\\ &= 2E[XY]-65.2\end{aligned}$$

$$E[XY] = (8+65.2)/2 = 36.6$$

Finally, $\text{Cov}(C_1, C_2) = 2.2(36.6)-71.72 = 8.8$