108. Solution: A

The joint density of T_1 and T_2 is given by

$$f\left(t_1,t_2\right)=e^{-t_1}e^{-t_2} \quad , \quad t_1>0 \quad , \quad t_2>0$$

Therefore,

$$\Pr\left[X\le x\right]=\Pr\left[2T_1+T_2\le x\right]$$

$$=\int_0^x\int_0^{\frac{1}{2}(x-t_2)}e^{-t_1}e^{-t_2}\,dt_1dt_2=\int_0^x e^{-t_2}\left[-e^{-t_1}\Big|_0^{\frac{1}{2}(x-t_2)}\right]dt_2$$

$$=\int_0^x e^{-t_2}\left[1-e^{-\frac{1}{2}x+\frac{1}{2}t_2}\right]dt_2=\int_0^x\left(e^{-t_2}-e^{-\frac{1}{2}x}e^{-\frac{1}{2}t_2}\right)dt_2$$

$$=\left[-e^{-t_2}+2e^{-\frac{1}{2}x}e^{-\frac{1}{2}t_2}\right]\Big|_0^x=-e^{-x}+2e^{-\frac{1}{2}x}e^{-\frac{1}{2}x}+1-2e^{-\frac{1}{2}x}$$

$$=1-e^{-x}+2e^{-x}-2e^{-\frac{1}{2}x}=1-2e^{-\frac{1}{2}x}+e^{-x} \quad , \quad x>0$$

It follows that the density of X is given by

$$g\left(x\right)=\frac{d}{dx}\left[1-2e^{-\frac{1}{2}x}+e^{-x}\right]=e^{-\frac{1}{2}x}-e^{-x} \quad , \quad x>0$$