109. Solution: B

Let

u be annual claims,
v be annual premiums,
$g(u, v)$ be the joint density function of U and V,
$f(x)$ be the density function of X, and
$F(x)$ be the distribution function of X.

Then since U and V are independent,

$$g(u,v)=\left(e^{-u}\right)\left(\frac{1}{2}e^{-v/2}\right)=\frac{1}{2}e^{-u}e^{-v/2} \quad , \quad 0<u<\infty \quad , \quad 0<v<\infty$$

and

$$F(x)=\Pr[X\le x]=\Pr\left[\frac{u}{v}\le x\right]=\Pr[U\le Vx]$$

$$=\int_0^\infty\int_0^{vx} g(u,v)\,du\,dv=\int_0^\infty\int_0^{vx}\frac{1}{2}e^{-u}e^{-v/2}\,du\,dv$$

$$=\int_0^\infty -\frac{1}{2}e^{-u}e^{-v/2}\Big|_0^{vx}\,dv=\int_0^\infty\left(-\frac{1}{2}e^{-vx}e^{-v/2}+\frac{1}{2}e^{-v/2}\right)dv$$

$$=\int_0^\infty\left(-\frac{1}{2}e^{-v(x+1/2)}+\frac{1}{2}e^{-v/2}\right)dv$$

$$=\left[\frac{1}{2x+1}e^{-v(x+1/2)}-e^{-v/2}\right]_0^\infty=-\frac{1}{2x+1}+1$$

Finally, $f(x)=F'(x)=\dfrac{2}{(2x+1)^2}$