

110. Solution: C

Note that the conditional density function

$$f\left(y \mid x = \frac{1}{3}\right) = \frac{f(1/3, y)}{f_x(1/3)}, \quad 0 < y < \frac{2}{3},$$

$$f_x\left(\frac{1}{3}\right) = \int_0^{2/3} 24(1/3)y \, dy = \int_0^{2/3} 8y \, dy = 4y^2 \Big|_0^{2/3} = \frac{16}{9}$$

It follows that $f\left(y \mid x = \frac{1}{3}\right) = \frac{9}{16} f(1/3, y) = \frac{9}{2} y$, $0 < y < \frac{2}{3}$

Consequently, $\Pr[Y < X \mid X = 1/3] = \int_0^{1/3} \frac{9}{2} y \, dy = \frac{9}{4} y^2 \Big|_0^{1/3} = \frac{1}{4}$