111. Solution: E

\[
\Pr[1 < Y < 3 | X = 2] = \int_{1}^{3} \frac{f(2, y)}{f_X(2)} dy
\]

\[
f(2, y) = \frac{2}{4(2-1)} y^{(4-1)/2-1} = \frac{1}{2} y^{-3}
\]

\[
f_X(2) = \left[ \int_{2}^{\infty} y^{-3} dy \right] = \frac{1}{4}
\]

Finally, \( \Pr[1 < Y < 3 | X = 2] = \int_{1}^{3} \frac{1}{2} y^{-3} \left[ y = 1 - \frac{1}{9} = \frac{8}{9} \right] \)

112. Solution: D

We are given that the joint pdf of X and Y is \( f(x, y) = 2(x+y), 0 < y < x < 1 \).

Now \( f_X(x) = \int_{0}^{x} (2x + 2y) dy = \left[ 2xy + y^2 \right]_{0}^{x} = 2x^2 + x^2 = 3x^2, 0 < x < 1 \)

so \( f(y|x) = \frac{f(x, y)}{f_X(x)} = \frac{2(x+y)}{3x^2} = \left( \frac{1 + \frac{y}{x}}{x} \right), 0 < y < x \)

\( f(y|x = 0.10) = \frac{2}{3} \left[ \frac{1}{0.1} + \frac{y}{0.01} \right] = \frac{2}{3} [10+100y], 0 < y < 0.10 \)

\( P[Y < 0.05|X = 0.10] = \int_{0}^{0.05} \frac{2}{3} [10+100y] dy = \left[ \frac{20}{3} y + \frac{100}{3} y^2 \right]_{0}^{0.05} = \frac{1}{3} + \frac{1}{12} = \frac{5}{12} = 0.4167 \).

113. Solution: E

Let

- \( W \) = event that wife survives at least 10 years
- \( H \) = event that husband survives at least 10 years
- \( B \) = benefit paid
- \( P \) = profit from selling policies

Then \( \Pr[H] = P[H \cap W] + \Pr[H \cap W^c] = 0.96 + 0.01 = 0.97 \)

and

\[
\Pr[W | H] = \frac{\Pr[W \cap H]}{\Pr[H]} = \frac{0.96}{0.97} = 0.9897
\]

\[
\Pr[W^c | H] = \frac{\Pr[H \cap W^c]}{\Pr[H]} = \frac{0.01}{0.97} = 0.0103
\]

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