

112. Solution: D

We are given that the joint pdf of X and Y is $f(x,y) = 2(x+y)$, $0 < y < x < 1$.

$$\text{Now } f_x(x) = \int_0^x (2x + 2y) dy = [2xy + y^2]_0^x = 2x^2 + x^2 = 3x^2, 0 < x < 1$$

$$\text{so } f(y|x) = \frac{f(x,y)}{f_x(x)} = \frac{2(x+y)}{3x^2} = \frac{2}{3} \left(\frac{1}{x} + \frac{y}{x^2} \right), 0 < y < x$$

$$f(y|x = 0.10) = \frac{2}{3} \left[\frac{1}{0.1} + \frac{y}{0.01} \right] = \frac{2}{3} [10 + 100y], 0 < y < 0.10$$

$$P[Y < 0.05 | X = 0.10] = \int_0^{0.05} \frac{2}{3} [10 + 100y] dy = \left[\frac{20}{3} y + \frac{100}{3} y^2 \right]_0^{0.05} = \frac{1}{3} + \frac{1}{12} = \frac{5}{12} = 0.4167.$$