115. Solution: A

Let $f_1(x)$ denote the marginal density function of X. Then

$$f_1(x) = \int_x^{x+1} 2x\,dy = 2xy\,\big|_x^{x+1} = 2x(x+1-x) = 2x \quad , \quad 0 < x < 1$$

Consequently,

$$f(y|\,x) = \frac{f(x,y)}{f_1(x)} = \begin{cases} 1 & \text{if: } \; x < y < x+1 \\ 0 & \text{otherwise} \end{cases}$$

$$E[Y|\,X] = \int_x^{x+1} y\,dy = \frac{1}{2}y^2\,\big|_x^{x+1} = \frac{1}{2}(x+1)^2 - \frac{1}{2}x^2 = \frac{1}{2}x^2 + x + \frac{1}{2} - \frac{1}{2}x^2 = x + \frac{1}{2}$$

$$E\left[Y^2|\,X\right] = \int_x^{x+1} y^2\,dy = \frac{1}{3}y^3\,\big|_x^{x+1} = \frac{1}{3}(x+1)^3 - \frac{1}{3}x^3$$

$$= \frac{1}{3}x^3 + x^2 + x + \frac{1}{3} - \frac{1}{3}x^3 = x^2 + x + \frac{1}{3}$$

$$\text{Var}[Y|\,X] = E\left[Y^2|\,X\right] - \left\{E[Y|\,X]\right\}^2 = x^2 + x + \frac{1}{3} - \left(x + \frac{1}{2}\right)^2$$

$$= x^2 + x + \frac{1}{3} - x^2 - x - \frac{1}{4} = \frac{1}{12}$$