

First, find the conditional probability function of  $N_2$  given  $N_1 = n_1$ :  $p_{2|1}(n_2 | n_1) = \frac{p(n_1, n_2)}{p_1(n_1)}$ ,

where  $p_1(n_1)$  is the marginal probability function of  $N_1$ . To find the latter, sum the joint probability function over all possible values of  $N_2$  obtaining

$$p_1(n_1) = \sum_{n_2=1}^{\infty} p(n_1, n_2) = \frac{3}{4} \left( \frac{1}{4} \right)^{n_1-1} \sum_{n_2=1}^{\infty} e^{-n_1} (1 - e^{-n_1})^{n_2-1} = \frac{3}{4} \left( \frac{1}{4} \right)^{n_1-1},$$

since  $\sum_{n_2=1}^{\infty} e^{-n_1} (1 - e^{-n_1})^{n_2-1} = 1$  as the sum of the probabilities of a geometric random variable. The

conditional probability function is

$$p_{2|1}(n_2 | n_1) = \frac{p(n_1, n_2)}{p_1(n_1)} = e^{-n_1} (1 - e^{-n_1})^{n_2-1},$$

which is the probability function of a geometric random variable with parameter  $p = e^{-n_1}$ . The mean of this distribution is  $1/p = 1/e^{-n_1} = e^{n_1}$ , and becomes  $e^2$  when  $n_1 = 2$ .