138. Key: D

Suppose the component represented by the random variable $X$ fails last. This is represented by the triangle with vertices at (0, 0), (10, 0) and (5, 5). Because the density is uniform over this region, the mean value of $X$ and thus the expected operational time of the machine is 5. By symmetry, if the component represented by the random variable $Y$ fails last, the expected operational time of the machine is also 5. Thus, the unconditional expected operational time of the machine must be 5 as well.

139. Key: B

The unconditional probabilities for the number of people in the car who are hospitalized are 0.49, 0.42 and 0.09 for 0, 1 and 2, respectively. If the number of people hospitalized is 0 or 1, then the total loss will be less than 1. However, if two people are hospitalized, the probability that the total loss will be less than 1 is 0.5. Thus, the expected number of people in the car who are hospitalized, given that the total loss due to hospitalizations from the accident is less than 1 is

$$\frac{0.49}{0.49 + 0.42 + 0.09 \cdot 0.5} \cdot 0 + \frac{0.42}{0.49 + 0.42 + 0.09 \cdot 0.5} \cdot 1 + \frac{0.09 \cdot 0.5}{0.49 + 0.42 + 0.09 \cdot 0.5} \cdot 2 = 0.534$$

140. Key: B

Let $X$ equal the number of hurricanes it takes for two losses to occur. Then $X$ is negative binomial with “success” probability $p = 0.4$ and $r = 2$ “successes” needed.

$$P[X = n] = \binom{n-1}{r-1} p^r (1-p)^{n-r} = \frac{n-1}{2-1} (0.4)^2 (1-0.4)^{n-2} = (n-1)(0.4)^2(0.6)^{n-2}, \text{ for } n \geq 2.$$ 

We need to maximize $P[X = n]$. Note that the ratio

$$\frac{P[X = n+1]}{P[X = n]} = \frac{n(0.4)^2(0.6)^{n-1}}{(n-1)(0.4)^2(0.6)^{n-2}} = \frac{n}{n-1} (0.6).$$

This ratio of “consecutive” probabilities is greater than 1 when $n = 2$ and less than 1 when $n \geq 3$. Thus, $P[X = n]$ is maximized at $n = 3$; the mode is 3.